S–6291

Sub. Code
23MMA1C1

M.Sc. DEGREE EXAMINATION, APRIL 2025

First Semester

Mathematics

ALGEBRAIC STRUCTURES

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define the normalizer of a in group G.

2. Write down the class equation of G.

3. When will you say that a group G is said to be solvable?

4. Define R -module. Give an example.

5. When will you say that the linear transformations are said to be similar?

6. Define the index of nilpotence of T.

7. Define the Jordan form of T.

8. Define the companion matrix of $f(x)$.

9. For all $A \in F_n$, prove that $(A') = A$.

10. Define the following transformations :

(a) Unitary; (b) Skew -Hermitian.

Part B $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) If $O(G) = p^2$ where p is a prime number, then prove that G is abelian.

Or

(b) If p is a prime numer, show that in s_p there are $(p-1)!+1$ elements x satisfying $x^p = e$.

12. (a) Suppose that G is the internal direct product of $N_1, N_2, ..., N_n$. Prove that for $i \neq j$, $N_i \cap N_j = (e)$. Also prove $ab = ba$ if $a \in N_i$, $b \in N_j$.

Or

(b) Let R be a Euclidean ring. Prove that any finitely generated R-module, M is the direct sum of a finite number of cyclic submodules.

13. (a) If V is n-dimensional over F and if $T \in A(V)$ has all its characteristic roots in F, then prove that T satisfies a polynomial of degree n over F.

Or

(b) Show that two nilpotent linear transformations are similar if and only if they have the same invariants.

14. (a) Prove that the matrix $\begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 1 & 1 & 0 \end{pmatrix}$ is nilpotent, and find its invariants.

Or

(b) If T in $A_F(V)$ has as minimal polynomial $p(x) = q(x)^e$, where $q(x)$ is a monic, irreducible polynomial in $F(x)$, then prove that a basic of V over F can be found in which the matrix of T is of the form

$$\begin{pmatrix} C(q(x)^{e_1}) & & \\ & C(q(x)^{e_2}) & \\ & & C(q(x)^{er}) \end{pmatrix}$$

Where $e = e_1 \geq e_2 \geq \ldots\ldots\ldots\ldots \geq e_r$

15. (a) Prove that the linear transformation T on V is Unitary if and only if it takes on orthonormal basis of V into an orthonormal basis of V.

Or

(b) Determine the rank and signature of the following real quadratic form.

$$x_1^2 + x_1x_2 + 2x_1x_3 + 2x_2^2 + 4x_2x_3 + 2x_3^2 .$$

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. If p is a prime number and $p^{\alpha} / o(G)$, then prove that G has a subgroup of order p^{α}.

17. (a) Prove that G is solvable if and only if $G^{(k)} = (e)$ for some integer k.

    (b) Let $G = s_n$, where $n \geq 5$, then prove that $G^{(k)}$ for $k = 1, 2, ...$ contains every 3-cycle of S_n.

18. If $T \in A(V)$ has all its characteristic roots in F, then prove that there is a basis of V in which the matrix of T is triangular.

19. Show that the elements S and T in $A_F(V)$ are similar in $A_F(V)$ if and only if they have the same elementary divisors.

20. (a) If F is a field of characteristic 0, and if $T \in A_F(V)$ in such that $trT^i = 0$ for all $i \geq 1$, then prove that T is nilpotent.

    (b) Prove that $tr(A+B) = trA + trB$ and that for $\lambda \in F$, $tr(\lambda A) = \lambda trA$.

S–6292

Sub. Code
23MMA1C2

M.Sc. DEGREE EXAMINATION, APRIL 2025

First Semester

Mathematics

REAL ANALYSIS – I

(CBCS – 2023 onwards)

Time : 3 Hours | Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define the total variation of f on the interval $[a,b]$.

2. Define the following terms :

   (a) Absolutely convergent

   (b) Conditionally convergent.

3. Define a Riemann-Stieltjes sum of f with respect of α.

4. Assume that $\alpha \nearrow$ on $[a,b]$. Prove that $\underline{I}(f,\alpha) \leq \underline{I}(f,\alpha)$.

5. State the second mean-value theorem for Riemann-Stieltjes integrals.

6. Define the oscillation of f on T.

7. Describe the Cesaro Summable with an example.

8. State the Bernstein's theorem.

9. When will you say that a sequence $\{f_n\}$ is said to be uniformly bounded on S ?

10. State the Weierstrass M-test.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Let f be of bounded variation on $[a,b]$ and assume that $c \in (a,b)$. Prove that f is of bounded variation on $[a,c]$ and on $[c,b]$. Also prove $V_f(a,b) = V_f(a,c) + V_f(c,b)$.

Or

(b) Let $\sum a_n$ be an absolutely convergent series having sum s. Prove that every rearrangement of $\sum a_n$ also converges absolutely and has sum s.

12. (a) Assume that $f \in R(\alpha)$ on $[a,b]$ and α has a continuous derivative α' on $[a,b]$. Prove that the Riemann integral $\int_s^b f(x)\alpha'(x)\,dx$ exists. Also prove

$$\int_a^b f(x)\,d\alpha(x) = \int_a^b f(x)\alpha'(x)\,dx .$$

Or

(b) Assume that $\alpha \nearrow$ on $[a,b]$.

(i) If P' is finer than P, then prove that

$\cup(P', f, \alpha) \le \cup(P, f, \alpha)$ and

$L(P', f, \alpha) \ge L(P, f, \alpha)$

(ii) For any two partitions P_1 and P_2, prove that $L(P_1, f, \alpha) \le \cup(P_2, f, \alpha)$.

13. (a) Let α be of bounded variation of $[a,b]$ and assume that $f \in R(\alpha)$ on $[a,b]$. Prove that $f \in R(\alpha)$ on every subinterval $[c,d]$ of $[a,b]$.

Or

(b) State and prove second fundamental theorem of integral calculus.

14. (a) State and prove the Cauchy condition for product theorem.

Or

(b) Establish the Tauber theorem.

15. (a) Assume that $f_n \to f$ uniformly on S. If each f_n is continuous at a point c of S, then prove that the limit function f is also continuous at c.

Or

(b) State and prove Dirichlet's test for uniform convergence theorem.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Let f be of bounded variation on $[a,b]$. If $x \in (a,b)$, let $V(x) = V_j(a,x)$ and put $V(a) = 0$. Prove that every point of continuity of f is also a point of continuity of V. Also prove the converse is true.

17. Assume that $\alpha \nearrow$ on $[a,b]$. Prove that following three statements are equivalent :

(a) $f \in R(\alpha)$ on $[a,b]$.

(b) f satisfies Riemman's condition with respect to α on $[a,b]$.

(c) $\underline{I}(f,\alpha) = \bar{I}(f,\alpha)$.

18. State and prove the lebesgue's criterion for Riemann-integrability.

19. State and prove the Abel's limit theorem.

20. Let $\{f_n\}$ be a boundedly convergent sequence on $[a,b]$. Assume that each $f_n \in R$ on $[a,b]$, and that the limit function $f \in R$ on $[a,b]$. Assume also that there is a partition p of $[a,b]$, say $P = \{x_0, x_1, ...x_m\}$, such that, on every subinterval $[c,d]$ not containing any of the points x_k, the sequence $\{f_n\}$ converges uniformly to f. Prove that $\lim_{n\to\infty} \int_a^b f_n(t)\,dt = \int_a^b \lim_{n\to\infty} f_n(t)\,dt = \int_a^b f(t)\,dt$.

S–6293

Sub. Code
23MMA1C3

M.Sc. DEGREE EXAMINATION, APRIL 2025

First Semester

Mathematics

ORDINARY DIFFERENTIAL EQUATIONS

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Find all solutions of $y'' + 16y = 0$.

2. Determine whether the function $\phi_1(x) = x^2$ and $\phi_2(x) = 5x^2$, $x \in (-\infty, \infty)$ are linearly dependent or independent.

3. Show that the functions : $\phi_1(x) = 1$, $\phi_2(x) = x$, $\phi_3(x) = x^2$ are linearly independent on $-\infty < x < \infty$.

4. Using annihilator method find a particular solution of $y'' = x^2 + e^{-x} \sin x$.

5. State the uniqueness theorem of linear equations with variable coefficients.

6. With the usual notations prove that $P_n(-x) = (-1)^n P_n(x)$.

7. Write down the Laguerre equation.

8. Show that $\Gamma(n+1) = n\Gamma(n)$.

9. Using separation of variables, find the solution ϕ of $y' = 3y^{2/3}$.

10. Show by an example that a continuous function need not satisfy Lipschitz condition.

Part B $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) Prove that two solutions ϕ_1, ϕ_2 of $L(y) = 0$ are linearly independent on an interval I if and only if $W(\phi_1, \phi_2)(x) \neq 0$ for all x in I.

Or

(b) Find all solutions of $y'' - y' - 2y = e^{-x}$.

12. (a) Consider the equation $y'' - 4y' = 0$.

(i) Compute three linearly independent solutions

(ii) Compute the Wronskian of the solutions found in (i)

(iii) Find the solution ϕ satisfying $\phi(0) = 0, \phi'(0) = 1, \phi''(0) = 0$.

Or

(b) Show that if g has K-derivatives and r is a constant, $D^k (r^{rx} g) = e^{rx} (D + r)^k (g)$.

13. (a) Prove that there exist n linearly independent solutions of $L(y)=0$ on I.

Or

(b) Find two linearly independent solutions of the equation $(3x-1)^2 y''+(9x-3)y'-9y=0$ for $x>\frac{1}{3}$.

14. (a) Compute the indicial polynomial and their roots of the equation $x^2 y''+(x+x^2)y'-y=0$.

Or

(b) Show that $x^{1/2} J_{-1/2}(x)=\frac{\sqrt{2}}{\Gamma(1/2)} \cos x$.

15. (a) Let M, N be two real-valued functions which have continuous first partial derivatives on some rectangle

$R: |x-x_0| \leqq a, |y-y_0| \leq b$. Prove that the equation $M(x, y)+N(x, y)\, y'=0$ is exact in R if and only if $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$ in R.

Or

(b) Consider the initial value problem $y'=3x+1, y(0)=2$.

(i) Show that all the successive approximations $\phi_0, \phi_1, \ldots$ exist for all real x.

(ii) Compute the first-four approximations $\phi_0, \phi_1, \phi_2, \phi_3$ to the solutions.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Let ϕ be any solution of $L(y) = y'' + a_1 y' + a_2 y = 0$ on an interval I containing a point x_0. Prove that for all x in I, $\|\phi(x_0)\| e^{-k|x-x_0|} \leqq \|\phi(x)\| \leqq \|\phi(x_0)\| e^{k|x-x_0|}$ where $\|\phi(x)\| = \left[|\phi(x)|^2 + |\phi'(x)|^2\right]^{1/2}$, $k = 1 + |a_1| + |a_2|$.

17. Find the solution ψ of the equation $y''' + y'' + y' + y = 0$ which satisfies $\psi(0) = 0$, $\psi'(0) = 1$, $\psi''(0) = 0$.

18. (a) Show that the coefficient of x^n in $P_n(x)$ is $(2n)!/2^n (n!)^2$.

    (b) Prove that $\int_{-1}^{1} P_n(x) P_m(x)\, dx = 0$, $(n \neq m)$.

19. Derive the Bessel function of zero order of the first kind.

20. State and prove the existence theorem for the convergence of the successive approximations.

S–6294

Sub. Code
23MMA1E1

M.Sc. DEGREE EXAMINATION, APRIL 2025

First Semester

Mathematics

Elective : NUMBER THEORY AND CRYPTOGRAPHY

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Prove that $v_n^2 - Du_n^2 = 4Q^n$.

2. Write any two properties of greatest common divisor.

3. Define canonical factorization with an example.

4. If m is a natural number and p is prime, then prove that
$$(mip) = \begin{cases} P & if\ P/m \\ 1 & if\ P \times m \end{cases}$$

5. Define fermat numbers.

6. State chinese remainder theorem.

7. Define Dirchlet product.

8. Find the value of $\left(\dfrac{3}{11}\right)$.

9. What is meant by linear algebra modulo N?

10. Define discrete logarithm.

Part B $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) Find x, y such that $(87, 27) = 87x + 27y$.

Or

(b) Prove that $3 \mid n$ if and only if $3 \mid t_{10}(n)$.

12. (a) If n is composite, and if p is the least prime factor of n, then prove that $p \le \sqrt{n}$.

Or

(b) Prove that there exist infinitely many primes of the form $4k - 1$.

13. (a) Let a and b belong to set S. Let R be an equivalence relation on S. Then prove that aRb if and only if $[a]_R = [b]_R$.

Or

(b) Solve the congruence $x^3 + 2x + 2 \equiv 0 \pmod{49}$.

14. (a) State and prove Euclid-Euler theorem.

Or

(b) If p is an odd prime, x, y are integers such that $x \equiv y \pmod{p}$, and $|x| = |y| = 1$, then prove that $x = y$.

15. (a) Solve the following system of simultaneous congruences :

$x + 3y \equiv 1 \pmod{26}$

$7x + 9y \equiv 2 \pmod{26}$

Or

(b) Write down the application of public key cryptography.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. (a) State and prove the Euclid's algorithm.

(b) Prove that n is even if and only if its last decimal digit is even.

17. State and prove the fundamental theorem of arithemetic.

18. State the prove Wilson's lemma.

19. State and prove lemma of Gauss.

20. Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in M_2(Z/NZ)$ and set $D = ad - bc$ prove that the following are equivalent :

(a) g.c.d $(D, N) = 1$;

(b) A has an inverse matrix ;

(c) If x and y are not both O in Z/NZ, then $\begin{pmatrix} x \\ y \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \end{pmatrix}$;

(d) A gives a 1 - to -1 correspondence of $(Z/NZ)^2$ with itself.

S–6295

Sub. Code
23MMA1E2

M.Sc. DEGREE EXAMINATION, APRIL 2025

First Semester

Mathematics

Elective : GRAPH THEORY AND APPLICATIONS

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define the following terms :

   (a) Vertex-transitive graph

   (b) Edge - transitive graph

2. What is meant by a bond?

3. Define a block of a graph with an example.

4. Draw a graph which is both Eulerian and Hamiltonian.

5. Find the number of different perfect matching in K_n.

6. Draw 3-edge chromatic graph.

7. Define an independent set of a graph with an example.

8. When will you say that a critical graph is block?

9. Embed K_{33} on mobius band.

10. How many orientations does a sample graph G have?

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Show that if a k-regular bipartite graph with $k>0$ has bipartition (x,y), then $|x|=|y|$.

Or

(b) Define a cut vertex with an illustration. Also prove that a vertex v of a tree G is a cut vertex of G if and only if $d(v)>1$.

12. (a) Prove that a graph G with $v \geq 3$ is 2-connected if and only if any two vertices of G are connected by at least two internally - disjoint paths.

Or

(b) With the usual notations, prove that $C(G)$ is well defined.

13. (a) State and prove the Halls' theorem.

Or

(b) If G is a bipartite graph, then prove that $x' = \Delta$.

14. (a) Prove that a set $S \subseteq V$ is an independent set of G if and only if $V-S$ is a covering of G. Also prove $\alpha + \beta = v$.

Or

(b) If G is k-critical, then prove that $\delta = k-1$.

15. (a) Show that K_5 is nonplanar.

Or

(b) State and prove Euler's formula for a connected plane graph.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. With the usual notations, prove that $\tau(K_n) = n^{n-2}$.

17. Show that a nonempty connected graph is Eulerian if and only if it has no vertices of odd degree.

18. State and prove the Berge theorem.

19. Prove that for any positive integer k, there exists a $k-$ chromatic graph containing no triangle.

20. If D is strict and $\min\{\delta^-, \delta^+\} \geq \upsilon/2 > 1$, then prove that D contains a directed Hamilton cycle.

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S–6298

Sub. Code
23MMA1E5

M.Sc. DEGREE EXAMINATION, APRIL 2025

First Semester

Mathematics

Elective — FUZZY SETS AND THEIR APPLICATIONS

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Section A $(10 \times 2 = 20)$

Answer **all** questions.

1. What will you say that a fuzzy set is said to be normal?

2. Define a fuzzy equivalence relation.

3. Define a belief measure function.

4. What is meant by consonant?

5. Write any two axioms of measure of fuzziness.

6. Define a measure of confusion.

7. What is meant by fuzzy neural networks?

8. Write short notes on fuzzy dynamic systems.

9. What is meant by multiperson decision making?

10. Write short notes on multistage decision making.

Section B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Compute the scalar cardinalities for each of the following fuzzy sets :

(i) $C(x) = \frac{x}{x+1}$ for $x \in \{0,1,2,....,10\} = x$;

(ii) $D(x) = \frac{(1-x)}{10}$ for $x \in \{0,1,2,...,10\} = x$.

Or

(b) Let $M_P = \begin{bmatrix} 0.3 & 0.5 & 0.8 \\ 0 & 0.7 & 1 \\ 0.4 & 0.6 & 0.5 \end{bmatrix}$ and $M_Q = \begin{bmatrix} 0.9 & 0.5 & 0.7 & 0.7 \\ 0.3 & 0.2 & 0 & 0.9 \\ 1 & 0 & 0.5 & 0.5 \end{bmatrix}$. Draw the sagittal diagram and also find $P \circ Q$.

12. (a) Let $x = \{a,b,c,d\}$. Given the basic assignment $m(\{a,b,c,d\}) = 0.5, m(\{a,b,d\}) = 0.2$ and $m(x) = 0.3$, Determine the corresponding belief and plausibility measures.

Or

(b) Prove that every possibility measure π on $\wp(x)$ can be uniquely determined by a possibility distribution function $r : x \to [0, 1]$ Via the formula $\pi(A) = \max_{x \in A} r(x)$ for each $A \in \wp(x)$.

13. (a) State and prove Gibbs' theorem.

Or

(b) Let m_x and m_y be marginal basic assignments on sets x and y, respectively and let m be a joint basic assignment on $x \times y$ such that $m(A \times B) = m_x(A) \cdot m_y(B)$ for all $A \in \mathfrak{P}(x)$ and $B \in \mathfrak{P}(y)$. Prove that $E(m) = E(m_x) + E(m_y)$.

14. (a) Enumerate the methods of defuzzification.

Or

(b) Formulate reasonable fuzzy inference rules for an air-conditioning fuzzy control system.

15. (a) Describe the concept of individual decision making in all details.

Or

(b) Explain the fuzzy ranking method.

Section C (3 × 10 = 30)

Answer any **three** questions.

16. (a) Let A, B be fuzzy sets defined on a universal set x. Prove that $|A| + |B| = |A \cup B| + |A \cap B|$.

(b) Order the fuzzy sets defined by the following membership functions (assuming $x \geq 0$) by the inclusion (subset) relation :

$$A(x) = \frac{1}{1+10x}, B(x) = \left(\frac{1}{1+10x}\right)^{\frac{1}{2}},\ C(x) = \left(\frac{1}{1+10x}\right)^{2}.$$

17. Show that a belief measure Bel on a finite power set $\mathfrak{P}(x)$ is a probability measure if and only if its basic assignment m is given by $m(\{x\}) =$ Bel $(\{x\})$ and $m(A) = 0$ for all subsets of x that are not singletons.

18. Prove that the function $I(N) = \log_2 N$ is the only function that satisfies axioms $I(N \cdot M) = I(N) + I(M)$ for all $N, M \in \mathbb{N}$ through $I(2) = 1$.

19. Find a fuzzy automation with $x = \{x_1, x_2, x_3\}$, $y = \{y_1, y_2, y_3\}$, $z = \{z_1, z_2, z_3, z_4\}$ whose output relations R and state transition relation S are defined, respectively, by the matrix

$$R = \begin{array}{c} \\ z_1 \\ z_2 \\ z_3 \\ z_4 \end{array} \begin{array}{c} \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 1 & 0.3 \end{bmatrix} \end{array}$$

and the three dimensional array

$$S = \begin{array}{c} \\ z_1 \\ z_2 \\ z_3 \\ z_4 \end{array} \begin{array}{c} \begin{array}{cccc} z_1 & z_2 & z_3 & z_4 \end{array} \\ \begin{bmatrix} 0 & 0.4 & 0.2 & 1 \\ 0.3 & 1 & 0 & 0.2 \\ 0.5 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \quad \begin{array}{c} \\ z_1 \\ z_2 \\ z_3 \\ z_4 \end{array} \begin{array}{c} \begin{array}{cccc} z_1 & z_2 & z_3 & z_4 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0.2 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0.3 & 0 & 0.6 \end{bmatrix} \end{array}$$

20. Assume that each individual of a group of five judges has a total preference ordering $P_i (i \in \mathbb{N}_5)$ on four figure skaters a, b, c, d. The orderings are : $P_1(a, b, c, d)$, $P_2 = (a, c, d, b)$, $P_3 = (b, a, c, d)$, $P_4 = (a, d, b, c)$. Use fuzzy multiperson decision making to determine the group decision.

S–6299

Sub. Code
23MMA1E6

M.Sc. DEGREE EXAMINATION, APRIL 2025

First Semester

Mathematics

Elective – DISCRETE MATHEMATICS

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. What is meant by formal proof and valid argument?

2. Define quantifiers with an example.

3. Find the recurrence relation and basis for the sequence $(1, 3, 3^2, \cdots\cdots)$.

4. Show that $f(x) = x/2$ is partial recursive.

5. Draw the Hasse diagram of D(45).

6. List the uses of Karnaugh map.

7. Define the Hamming distance with an example.

8. Show that the $(m, m+1)$ parity check code can detect one error.

9. How many different bit strings of length seven are there?

10. Let n be a nonnegative. Prove that $\sum_{k=0}^{n} 2^k \binom{n}{k} = 3^n$.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Obtain a disjunctive normal form of $\neg(P \vee Q) \leftrightarrow (P \wedge Q)$.

Or

(b) Show that $(\forall x)\,(P(x)) \vee (\forall x)\,(Q(x)) \rightarrow (\forall x)\,(P(x) \vee Q(x))$ is logically valid.

12. (a) Find the generating function of fibonacci sequence.

Or

(b) Show that $f(x, y) = x^y$ is primitive recursive.

13. (a) Prove that every chain is a lattice.

Or

(b) Consider the Boolean function

$f(x_1, x_2, x_3) = ((x_1 + x_2) + (x_1 + x_3)).\quad x_1 \cdot \bar{x}_2$ simplify this function and draw the circuit gate diagram for it.

14. (a) Prove that an (m,n) encoding function $e: B^m \to B^n$ can detect k or fewer errors if and only if its minimum distance is at least $k+1$.

Or

(b) Define a group code. Also prove that $(m, m+1)$ parity check code $e: B^m \to B^{m+1}$ is a group code.

15. (a) How many poker hands of five cards can be dealt from a standard deck of 52 cards? Also, how many ways are there to select 47 cards from a standard deck of 52 cards?

Or

(b) Write down the algorithm for generating the next r-combination in lexicographic order. Also find the next 4-combination of the set $\{1,2,3,4,5,6\}$ after $\{1,2,5,6\}$.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. Give an argument which will establish the validity of the following inference:

All intergers are rational numbers.

Some intergers are powers of 2.

Therefore, some rational numbers are power of 2.

17. Solve the recurrence relation.

$s(k) - 4s(k-1) - 11s(k-2) + 30s(k-3) = 0$, $s(0) = 0$, $s(1) = -35$, $s(2) = -85$.

18. Simplify

$f(a,b,c,d,e) = \sum(0,1,3,8,9,13,14,15,16,17,19,24,25,27,31)$.

19. Suppose e is an (m, n) encoding function and d is a maximum likelihood decoding function associated with e. Prove that (e, d) can correct k or fewer errors if and only if the minimum distance of e is least $2k+1$.

20. (a) State and prove the Pigeonhole principle and the generalized Pigeonhole principle.

    (b) What is the minimum number of students required in a discrete mathematics class to be sure that at least six will receive the same grade, if there are five possible grades, A, B, C, D and F?

S–6300

Sub. Code
23MMA2C1

M.Sc. DEGREE EXAMINATION, APRIL 2025

Second Semester

Mathematics

ADVANCED ALGEBRA

(CBCS – 2023 onwards)

Time : 3 Hours | Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** the questions

1. Define a field extension and give an example.

2. What do you mean by algebraic of degree n over F ?

3. Define a root of multiplicity.

4. What do you mean by splitting field?

5. Define group of automorphisms of k relative to F .

6. Prove that the fixed field of G is a subfield of k .

7. Define a solvable group.

8. Define a finite field and give an example.

9. Prove that for all $x, y \in Q$, $N(x.y) = N(x).N(y)$.

10. State the Lagrange Identify.

Part B (5 × 5 = 25)

Answer **all** the questions choosing either (a) or (b).

11. (a) Prove that the elements in K, which are algebraic over F form a sub field of K.

Or

(b) State and prove the transitivity property of algebraic extensions.

12. (a) Prove that τ^* defines an isomorphism of $F[x]$ onto $F'[t]$ with the property that $\tau^*(\alpha) = \alpha'$, for every $\alpha \in F$.

Or

(b) Show that the polynomial $f(x) \in F[x]$ has a multiple root if and only if $f(x)$ and $f'(x)$ have a non–trivial common factor.

13. (a) If K is a finite extension of F, then prove that $G(K,F)$ is a finite group and its order $O[G(K,F)]$ satisfies $O[G(K,F)] \leq [K:F]$.

Or

(b) Let F_0 be the field of rational numbers and let $K = F_0\left[\sqrt[3]{2}\right]$, then find fixed field of $G(K, F_0)$.

14. (a) Prove that for every prime number p and every positive integer m, the exists a field having p^m elements.

Or

(b) Show that if F is a finite field and $\alpha \neq 0, \beta \neq 0$ are two elements of F, then we can find elements a and b in F, such that $1 + \alpha a^2 + \beta . b^2 = 0$.

15. (a) Prove that S_n is not solvable for $n \geq 5$.

Or

(b) If C is the field of complex numbers and suppose that the division ring D is algebraic over C, then prove that $D = C$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Prove that the number e is transcendental.

17. Prove that if F is of characteristics O and if 'a' and 'b' are algebraic over F, then there exists and element $C \in F(a,b)$, such that $F(a,b) = F(c)$.

18. Show that K is the normal extension of F if and only if K is the splitting field of some polynomial over F.

19. State and prove the wedderburn theorem.

20. If $p(x) \in F[x]$ is solvable by radicals over F, then prove that the Galois group over F of $p(x)$ is a solvable group.

S–6301

Sub. Code
23MMA2C2

M.Sc. DEGREE EXAMINATION, APRIL 2025

Second Semester

Mathematics

REAL ANALYSIS – II

(CBCS – 2023 onwards)

Time : 3 Hours   Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Prove that for any set A there exists a measurable set E containing A and such that $m^*(A) = M(E)$.

2. Define essentially bounded.

3. Define Riemann Integrable on $(-\infty, \infty)$.

4. Define Lebesgue Integral.

5. Define Dirichlet Kernel.

6. State the Jordon's test.

7. Define Total derivative.

8. Write the Taylor's formula.

9. Define Jacobian determinant.

10. State the second derivative test for extrema.

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) Show that there exists uncountable sets of zero measure.

Or

(b) Prove that every interval is measurable.

12. (a) Show that $\lim \int_0^{\infty} \frac{dx}{\left(1+\frac{x}{n}\right)^n x^{\frac{1}{n}}} = 1$.

Or

(b) Let f and g be non negative measurable function then prove that $\int t dx + \int g dx = \int (f+g)\, dx$.

13. (a) Let $f \in L(I)$, then prove that for each real β,

$$\lim_{\infty \to +\infty} \int_I f(t) \sin(\alpha t + \beta) dt = 0$$

Or

(b) State and prove Riesz - Fischer theorem.

14. (a) Prove that if f is differentiable at c, then f is continuous at c.

Or

(b) State the prove mean value theorem.

15. (a) Assume that $f = (f_1 ..... f_n)$ has continuous partial derivatives $D_j f_i$ on an open set S in $\mathbb{R}^n$, and that the Jacobian determinant $J_f(a) \neq 0$, for some point a in S. Then prove that there is an n-ball $B(a)$ on which f is one to one.

Or

(b) Let f be a real valued function with continuous second order partial derivatives at a stationary point a in $\mathbb{R}^2$. Let $A = D_{1,1} f(a)$ $B = D_{1,2} f(a), C = D_{2,2} f(a)$ and let $D = \det\begin{bmatrix} A & B \\ B & C \end{bmatrix} = AC - B^2$, then prove the following.

(i) If $\Delta > 0$ and $A > 0$, f has a relative minimum

(ii) If $\Delta > 0$ and $A < 0$, f has a relative maximum.

(iii) If $\Delta < 0$, f has a saddle point at a.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. Prove that the class M is a $\sigma-$ algebra.

17. Let f be a bounded function defined on the finite interval $[a,b]$ then prove that f is Riemann integrable over $[a,b]$ if and only if is continuous a.e.

18. Let f be real valued and continuous on a compact interval $[a,b]$ then prove that for every $\in \geq 0$, there is a polynomial p such that $|f(x) - p(x)| < \in$ for every x in (a,b).

19. Assume that g is differentiable at a, with total derivative $g'(a)$. Let $b = g(a)$ and assume that f is differentiable at b, with total derivative $f'(b)$. Then prove that the composite function $f = f \circ g$ is differentiable at a, and the total derivative $h'(a)$ is given by $h'(a) = f'(b) \circ g'(a)$, the composition of the linear function $f'(b)$ and $g'(a)$.

20. State the prove the Inverse function theorem.

S–6302

Sub. Code
23MMA2C3

M.Sc. DEGREE EXAMINATION, APRIL 2025.

Second Semester

Mathematics

PARTIAL DIFFERENTIAL EQUATIONS

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** the questions.

1. What are the assumptions made by deriving the partial differential equation in elastic medium?

2. Classify the PDE $u_{xx} - 4u_{xy} + 4u_{yy} = e^y$.

3. State the Well-Posed problem.

4. Write the one-dimensional wave equation spherical polar co-ordinates.

5. Define the mixed boundary condition.

6. For the plucked string problem, what are the initial and boundary conditions.

7. Define the Neumann Boundary value problem.

8. State the mean value theorem.

9. What do you mean by the method of images?

10. Define a Dirac delta function.

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) Write all assumptions in the derivation of the PDE for the vibrating membrane.

Or

(b) Classify and transform the PDE $u_{xx} + u_{yy} + u_{xy} + u_x = 0$ into canonical form.

12. (a) Find the solution of the characteristic initial value problem $u_{tt} = c^2 u_{xx}$; $u(x,t) = f(x)$ on $x + ct = 0$ $u(x,t) = g(x)$ on $x - ct = 0$ when $f(0) = g(0)$.

Or

(b) Determine the solution of the non-homogeneous wave equation $u_{xx} - u_{yy} = 1$, $u(x,0) = \sin x$ and $u_y(x,0) = x$.

13. (a) Explain the struck string problem for finding the solution of the vibration of a stretched string.

Or

(b) State and prove the uniqueness theorem on heat conduction problem.

14. (a) State and prove the maximum principle theorem.

Or

(b) Derive the solution of the Dirichlet problem for a rectangle.

15. (a) Show that the Green's function is symmetric.

Or

(b) Determine the solution of the Dirichlet problem $\nabla^2 u = h$ in D, $u = f$ on B, by the method of Green's function.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Classify the PDE $yu_{xx} + 3yu_{xy} + 3u_x = 0$ into the canonical form and hence find the general solution.

17. Solve the Cauchy problem of an infinite string with the initial conditions $u_{tt} = c^2 u_{xx}$, $u(x,0) = f(x)$, $u_t(x,0) = g(x)$.

18. Establish the solution of the one-dimensional wave equation with suitable initial and boundary conditions using the method of separation of variables.

19. Derive the poisson integral formula for the interior of a circle.

20. Determine the Green's function solution of the Dirichlet problem involving the Helmholtz operator.

S–6304

Sub. Code
23MMA2E2

M.Sc. DEGREE EXAMINATION, APRIL 2025

Second Semester

Mathematics

Elective – MATHEMATICAL STATISTICS

(CBCS – 2023 onwards)

Time : 3 Hours   Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. If C_1 and C_2 are events such that $C_1 \subset C_2$, then prove that $P(C_1) \leq P(C_2)$.

2. Define the conditional probability.

3. Let the pmf $p(x)$ be positive at $x = -1, 0, 1$ and zero elsewhere. If $p(0) = \frac{1}{4}$, find $E(x^2)$.

4. Write down the Jacobian of the transformation.

5. If the mgf of a random variable X is $\left(\frac{1}{3} + \frac{2}{3}e^t\right)^5$, find $P(X = 2 \text{ or } 3)$.

6. Define a Chi-square distribution. Give an example.

7. State student's test distribution.

8. When will you say that statistic (T) is an unbiased estimator of θ?

9. Define a random sample.

10. Write down the accept-reject algorithm.

Part B $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) State and prove the Boole's inequality.

Or

(b) Let the space of the random variable X be $C = \{x : 0 < x < 10\}$ and let $P_x(C_1) = \frac{3}{8}$, where $C_1 = \{x : 1 < x < 5\}$. Show that $P_x(C_2) \leq \frac{5}{8}$, where $C_2 = \{X : 5 \leq x < 10\}$.

12. (a) Find the mean and variance of the following distribution. $f(x) = 6x(1-x)$, $0 < x < 1$, zero elsewhere.

Or

(b) Let X_1 and X_2 have the joint pdf $f(x_1, x_2) = \begin{cases} 2 & 0 < x_1 < x_2 < 1 \\ 0 & else\ where \end{cases}$. Find the marginal probability density functions.

13. (a) Define a Poisson distribution with an example. Let X have a Poisson distribution. If $P(X=1) = P(X=3)$, find the mode of the distribution.

Or

(b) Find the mean and variance of the β-distribution.

14. (a) If the independent variables X_1 and X_2 have means μ_1, μ_2 and variances σ_1^2, σ_2^2 respectively, show that the mean and variance of the product $Y = X_1X_2$ are μ_1, μ_2 and $\sigma_1^2\sigma_2^2 + \mu_1^2\sigma_2^2 + \mu_2^2\sigma_1^2$ respectively.

Or

(b) Establish F-distribution.

15. (a) Let $X_1, X_2, ..., X_n$ be a random sample from a continuous type distribution.

(i) Find $P(X_1 \le X_2)$, $P(X_1 \le X_2, X_1 \le X_3)$,.... $P(X_1 \le X_i, i = 2, 3, ...n)$.

(ii) Compute the mean and variance of Y if they exists.

Or

(b) Assume that the weight of cereal in a "10-ounce box" is $N(\mu, \sigma^2)$. To test $H_0 = \mu = 10.1$ against $H_1 : \mu > 10.1$ take a random sample of size $n = 16$ and observe that $\overline{x} = 10.4$ and $s = 0.4$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Cast a die a number of independent times until a six appears on the up side of the die.

(a) Find the pmf $p(x)$ of X, the number of casts needed to obtain that first six.

(b) Show that $\sum_{x=1}^{\infty} p(x) = 1$.

(c) Determine $P(X = 1, 3, 5, 7, ...)$

(d) Find the cdf $F(x) = P(X \le x)$.

17. (a) Suppose X_1 and X_2 are independent and that $E(u(X_1))$ and $E(v(X_2))$ exists. Prove that

$$E[u(X_1)v(X_2)] = E[u(X_1)]E[v(X_2)].$$

(b) Show that the random variables X_1 and X_2 with joint pdf

$$f(x_1, x_2) = \begin{cases} 12\, x_1, x_2(1-x_2)\, 0 < x_1 < 1, 0 < x_2 < 1 \\ 0 \quad \text{elsewhere} \end{cases}$$

are independent.

18. (a) Show that the graph of a p.d.f $N(\mu, \sigma^2)$ has points of inflection at $x = \mu - \sigma$ and $x = \mu + \sigma$.

(b) Let X be $N(0,1)$. Use the moment generating function technique to show that $Y = X^2$ is $\chi^2(1)$.

19. (a) State and prove central limit theorem.

(b) Establish weak law of large numbers.

20. (a) Discuss the problem of finding a confidence interval for the difference $\mu_1 - \mu_2$ between the two means of two normal distribution if the variances σ_1^2 and σ_2^2 are known but not necessarily equal.

(b) Suppose the random variable U ha a uniform $(0,1)$ distribution. Let F be a continuous distribution function. Prove that the random variable $X = F^{-1}(U)$ has distribution function F.

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S–6306

Sub. Code
23MMA2E4

M.Sc. DEGREE EXAMINATION, APRIL 2025.

Second Semester

Mathematics

Elective : CALCULUS OF VARIATIONS AND INTEGRAL EQUATIONS

(CBCS – 2023 onwards)

Time : 3 Hours — Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. What is the Euler-Lagrange equation for calculus of variation?

2. Define Poisson's equation.

3. Find the transversality condition for functionals of the form $v = \int_{x_0}^{x_1} A(x, y)\sqrt{1 + y^2}\, dx$.

4. Define strong extremum.

5. What is isoperimetric variational problem?

6. What is Newton's method for functional equations kantorovich?

7. Define separable Kernel.

8. Define resolvent Kernel for fredholm integral equation.

9. Find the value of c_0, c_1 and c_2 in the integral equation $g(s) = s + \lambda \int_0^1 \left[st + (st)^{1/2} \right] g(t)\, dt$.

10. Define volterra integral equation.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Find the extremal of the functional $v[y(x)] = \int_0^1 (1 + y'')^2\, dx$; $y(0) = 0$, $y'(0) = 1$, $y(1) = 1$, $y'(1) = 1$.

Or

(b) Derive the differential equation of free vibrations of a string.

12. (a) Find the broken-line extremals of the functional $v = \int_{x_0}^{x_2} y'^2 (1 - y')^2\, dx$.

Or

(b) Test for am extremum the functional $v[y(x)] = \int_0^a y'^3 dx$; $y(0) = 0$, $y(a) = b$, $a > 0$ and $b > 0$.

13. (a) Find the extremals of the isoperimetric problem $v[y(x)] = \int_{x_0}^{x_1} y'^2 dx$ given that $\int_{x_0}^{x_1} y\, dx = a$, where a is a constant.

Or

(b) Using the Ritz method, find and approximate solution of the problem of the minimum of the functional $v[y(x)] = \int_0^2 [y'^2 + y^2 + 2xy]\,dx$; $y(0) = y(2) = 0$ and compare it with the exact solution.

14. (a) Solve the integral equation $g(s) = f(s) + \lambda \int_0^1 (s+t)\, g(t)\, dt$ and find the eigen values.

Or

(b) Find the resolvent Kernel for the integral equation $g(s) = f(s) + \lambda \int_{-1}^1 (st + s^2 t^2)\, g(t)\, dt$.

15. (a) Solve the volterra equation $g(s) = 1 + \int_0^s st\; g(t)\, dt$.

Or

(b) Solve the integral equation $g(s) = s + \lambda \int_0^1 \left[st + (st)^{1/2}\right] g(t)\, dt$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Write the ostrogradsky equation for the functional $v[u(x,y,z)] = \iiint_D \left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 + 2u\, f(x,y,z) \right]$ $dxdydz$.

17. Test for an extremum the functional $\int_0^a \left(6y'^2 - y'^4 - yy'\right)dx$; $y(0) = 0$; $y(a) = b$; $a > 0$, $b > 0$.

18. Find a continuous solution of the equation $\Delta z = -1$ in the domain D, which is an isosceles triangle bounded by the straight line $y = \pm\frac{\sqrt{3}}{3}x$ and $x = b$.

19. Show that the intergral equation $g(s) = f(s) + \left(\frac{1}{\pi}\right) \int_0^{2\pi} [\sin(s+t)] g(t)\, dt$ possesses no solution for $f(s) = s$, but that is possesses infinitely many solutions when $f(s) = 1$.

20. State and prove Fredholm's third theorem.

S–6307

Sub. Code
23MMA2E5

M.Sc. DEGREE EXAMINATION, APRIL 2025

Second Semester

Mathematics

Elective : WAVELETS

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define discrete delta function.

2. Define translation of invariant.

3. Define the conjugate reflection of W.

4. Define first-stage wavelet basis for $l^2(\mathbb{Z}_N)$.

5. Define complete.

6. Prove that the trigonometric system is an orthonormal set in $l^2([-\pi, \pi))$.

7. Define inverse fourier transform on $L^2([-\pi, \pi))$.

8. Define complete orthonormal set in $l^2(\mathbb{Z})$.

9. Define support of f.

10. Define mother wavelet.

Part B $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) Prove that the set $\{E_o, ...E_{N-1}\}$ is an orthonormal basis for $l^2(\mathbb{Z}_N)$.

Or

(b) Suppose $T : l^2(\mathbb{Z}_N) \rightarrow l^2(\mathbb{Z}_N)$ is a linear transformation. Let $A_{T,E}$ be the matrix representing T is the standard basis E. If T is translation invariant, then prove that $A_{T,E}$ in circulant.

12. (a) Let $W \in l^2(\mathbb{Z}_N)$ then prove that $\{R_k w\}_{k=0}^{N-1}$ is an orthonormal basis for $l^2(\mathbb{Z}_N)$ if and only if $|\hat{w}(n)| = 1$ for all $n \in \mathbb{Z}_N$.

Or

(b) State and prove the folding lemma.

13. (a) Suppose H is a Hilbert space, $\{a_j\}_{a \in z}$ is an orthonormal set in H_1 and $z = z(j)_{j \in z} \in l^2(\mathbb{Z})$. Then prove that the series $\sum_{j \in z} (z(j))\, a_j$ converges in H and

$$\left\| \sum_{j \in z} z(j)\, a_j \right\|^2 = \sum_{j \in z} |\, z(j) \,|^2 .$$

Or

(b) Suppose H is a Hilbert space and $\{a_j\}_{j \in z}$ is an orthonormal set in H then prove that $\{a_j\}_{j \in z}$ is a complete orthonormal set H and only if $f = \sum_{j \in z} \langle f, a_j \rangle aj$ for all $f \in H$.

14. (a) Suppose $u \in l'(\mathbb{Z})$ and $\{R_{2k}u\}_{k\in z}$ is orthonormal is $l^2(\mathbb{Z})$. Define a sequence $v \in l'(\mathbb{Z})$ by $v(k) = (-1)^{k-1}\overline{u(1-k)}$. Then prove that $\{R_{2k}v\}_{k\in \mathbb{Z}} \cup \{R_{2k}v\}_{k\in \mathbb{Z}}$ is a complete orthonormal system in $l^2(\mathbb{Z})$.

Or

(b) Suppose $z \in l^2(\mathbb{Z})$ and $w \in l'(\mathbb{Z})$. Then prove that $z * w \in l^2(\mathbb{Z})$ and $\| z * w \| \leq \| w \|, \| z \|$.

15. (a) If $G : \mathbb{R} \to \mathbb{R}$ by define $G(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$. Then prove the following :

(i) $\int_R G(x)\, dx = 1$

(ii) There exists $c_1 > 0$ such that $G(x) \leq \frac{c_1}{(1+|x|)^2}$.

(iii) $\hat{G}(\xi) = e^{-\xi^2/2}$, or $\hat{G} = \sqrt{2\pi}G$.

Or

(b) State and prove the parseval's relation and plancherel's formula.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. If $z \in l^2(\mathbb{Z}_N)$ and $k \in \mathbb{Z}$. Then prove that for any $m \in \mathbb{Z}$, $(R_k z)^n(m) = e^{-2\pi imk/N}\hat{Z}(m)$.

17. Suppose $m \in \mathbb{N}$, $N = 2M$ and $w \in l^2(\mathbb{Z}_N)$. Then prove that $\{R_{2k}w\}_{k=0}^{M-1}$ is an orthonormal set with M elements it and only if $|\hat{w}(n)|^2 + |\hat{w}(n+m)|^2 = 2$ for $n = 0,1,..M-1$.

18. Suppose $f:[-\pi, \pi) \to \mathbb{C}$ is continuous and bounded, day $|f(\theta)| \le M$ for all θ if $\langle f, e^{in\theta} \rangle = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(\theta) e^{-in\theta} d\theta = 0$ for all $n \in \mathbb{Z}$, then prove that $f(\theta) = 0$ for all $\theta \in [-\pi, \pi)$.

19. Suppose $v, w \in l'(\mathbb{Z})$ and $z \in l^2(\mathbb{Z})$ then prove that following

(a) $(z * w)^n (\theta) = \hat{2}(\theta) \hat{w}(\theta)$ a.e.

(b) $z * w = w * z$

(c) $v * (w * z) = (v * w) * z$

20. Suppose $f \in L'(\mathbb{R})$ and $\{g_t\}_{t>0}$ is an approximate identity for some $c_1 > 0$. Then prove that, for every lebesgue point x of f, $\lim_{t \to 0^+} g_t * f(x) = f(x)$.

S–6308

Sub. Code
23MMA2E6

M.Sc. DEGREE EXAMINATION, APRIL 2025.

Second Semester

Mathematics

Elective – MACHINE LEARNING AND ARTIFICIAL INTELLIGENCE

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A ($10 \times 2 = 20$)

Answer **all** the questions.

1. Define inductive learning.

2. How do you design a checkers learning problem.

3. Define delta rule.

4. What is meant by genetic algorithm?

5. What are the advantages of Naive bayes?

6. Give the Baye's rule equation.

7. What is AI?

8. List the properties of task environments.

9. What is planning?

10. Define stochastic dominance.

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) What do you mean by well-posed learning problem? Explain the important features that are required to well define a learning problem.

Or

(b) What is the different between find-S and candidate elimination algorithm?

12. (a) Discuss the perceptron training rule.

Or

(b) Explain how to learn multiplayer networks using gradient descent algorithm.

13. (a) Explain Brute force MAP hypothesis learner. What is minimum description length principle?

Or

(b) Explain Bayesian belief network and conditional independence with an example.

14. (a) Define the following terms:

(i) Intelligence

(ii) Artificial intelligence

(iii) Agent

(iv) Rationality and

(v) Logical reasoning.

Or

(b) Explain Alpha-Beta pruning using example.

15. (a) Write forward state-space search algorithm.

Or

(b) Explain in detail about value iteration algorithm.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. Describe hypothesis space search in ID3 and contrast it with candidate elimination algorithm.

17. Explain in detail about perceptrons and its types.

18. Explain the concept of Bayes theorem with an example.

19. Is AI is a science (or) is it engineering? or Neither or both? Explain.

20. Discuss about hidden Markov model and give an example.

S–6309

Sub. Code
23MMA3C1

M.Sc. DEGREE EXAMINATION, APRIL 2025.

Third Semester

Mathematics

COMPLEX ANALYSIS

(CBCS – 2023 onwards)

Time : 3 Hours   Maximum : 75 Marks

Part A   ($10 \times 2 = 20$)

Answer **all** the questions.

1. Define the winding number.

2. Show that the function $\sin z$ have essential singularities at ∞.

3. When will you say that a differential $pdx + qdy$ is said to be locally exact in Ω?

4. Find the poles and residues of $\dfrac{1}{\sin^2 z}$.

5. Write down the Laplace's equations in polar and Cartesian forms.

6. State the Schwarz's formula.

7. Write short notes on the reflection principle.

8. State the Huruitz theorem.

9. Define an entrie function. Give an example.

10. State the Poisson-Jensen formula.

Part B $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) State and prove the Taylors theorem.

Or

(b) Establish the Schwarz lemma.

12. (a) (i) State the general form of Cauchy's theorem.

(ii) How many roots does the equation $z^7 - 2z^5 + 6z^3 - z + 1 = 0$ have in the disk $|z| < 1$?

Or

(b) State and prove the argument principle theorem.

13. (a) Derive the mean-value property.

Or

(b) State and prove the poisson formula.

14. (a) State and prove the Weierstrass's theorem on power series.

Or

(b) State and prove the Schwarz's theorem.

15. (a) With the usual notations, prove that

$$\frac{\pi^2}{\sin^2 \pi z} = \sum_{-\infty}^{\infty} \frac{1}{(z-n)^2}.$$

Or

(b) Define the Gamma function. Also prove that

(i) $\Gamma(z+1) = z\Gamma(z)$

(ii) $\Gamma(1/2) = \sqrt{\pi}$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. State and prove the Cauchy's representation formula. Deduce that $f^{n}(z)=\frac{n!}{2\pi i}\int_{C}\frac{f(r)dr}{(r-z)^{n+1}}$.

17. If $pdx+qdy$ is locally exact in Ω, then prove that $\int_{\gamma} pdx+qdy=0$ for every cycle $\gamma \sim 0$ in Ω.

18. Show that $\int_{0}^{\pi}\log\sin x dx=\pi\log(1/2)$.

19. Obtain the Laurent expansion $\sum_{n=-\infty}^{\infty}A_n(z-a)^n$ for the function $f(z)$ analytic in $R_1<|z-a|<R_2$.

20. State and prove the Hadamard's theorem.

S–6310

Sub. Code
23MMA3C2

M.Sc. DEGREE EXAMINATION, APRIL 2025

Third Semester

Mathematics

PROBABILITY THEORY

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Write down the properties of z.

2. Prove that the sum of the probability of any event A and its complement $\overline{A}$ is one.

3. Let X be the random variable from the normal distribution with density $f(x)=\frac{e^{-x^2/2}}{\sqrt{2\pi}}$. Find $E\left(X^2\right)$.

4. Find $E(X)$ for the Poisson distribution.

5. Define semi-invariants.

6. Write down the necessary and sufficient conditions for a function $\phi(t)$ to be a characteristics function.

7. What is meant by zero-one distribution?

8. Define rectangular distribution.

9. State the Kolmogorov inequality.

10. Define limit distribution function.

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) A deck cards contains 52 cards. Player G has been dealt 13 of them. Compute the probability that player G has

(i) any 3 face cards of the same face value

(ii) any 3 cards of the same face value from the 8 lowest dominations

(iii) at least three successive cards of any suit.

Or

(b) Let $\{A_n\}$, $n = 1, 2, \ldots,$ be a non increasing sequence of events and let A be their product. Prove that $P(A) = \lim_{n \to \infty} P(An)$.

12. (a) State and prove the Lapunov inequality.

Or

(b) Prove that $E(Xy) = E(X)E(Y)$.

13. (a) Find the characteristic function and the moments of a normal distribution.

Or

(b) Suppose two independent random variables X_1 and X_2 have Poisson distributions. Determine the characteristic function and the semi-invariants of $Z = X_1 - X_2$.

14. (a) The random variable X has the distribution $N(1:2)$. Find the probability that X is greater than 3 in absolute value.

Or

(b) Find the 1st and 2nd moments of polya distribution.

15. (a) Prove that the sequence $\{X_n\}$ is stochastically convergent to zero if and only if the sequence $\{F_n(x)\}$ satisfies the relation $\lim_{n\to\infty} F_n(x) = \begin{cases} 0 & for \quad x \le 0 \\ 1 & for \quad x > 0 \end{cases}$, where $F_n(x)$ $(n = 1, 2, \ldots)$ is the distribution function of the random variable X_n.

Or

(b) State and prove the Chebyshev's theorem.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. The probability function of the random vector (X, Y) is of the form $P(X = 0,\ 1 \le Y \le 2) = P(1 \le X \le 2,\ Y = 0) = \frac{1}{2}$

(a) Find the discontinuity points of the distribution function $F(x, y)$

(b) Check whether the plaints rectangle with vertices $(0,0), (2,0), \left(0, \frac{1}{2}\right)$ and $\left(2 \frac{1}{2}\right)$ is a continuity rectangle.

17. Prove that the equality $p^2 = 1$ is a necessary and sufficient condition for the relation $P(Y = aX + b) = 1$ to hold.

18. Let $F(x)$ and $\phi(t)$ denote respectively the distribution function and characteristic function of the random variable X. If $a+h$ and $a-h\,(h>0)$ are continuity points of the distribution function $F(x)$, prove that $F(a+h) - F(a-h) = \lim_{T\to\infty} \frac{1}{\pi} \int_{-T}^{T} \frac{\sin ht}{t} e^{-ita} \phi(t)\,dt$.

19. Derive the characteristic function of a Cauchy distribution.

20. State and prove the Levy-Cramer theorem.

S–6311

Sub. Code
23MMA3C3

M.Sc. DEGREE EXAMINATION, APRIL 2025

Third Semester

Mathematics

TOPOLOGY

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. What do you mean by basis for a topology on a set x?
2. Define the subspace topology. Give an example.
3. Can you conjecture what functions $f: R \to R$ are continuous when considered as maps from R to R_l?
4. Define the uniform topology. Give an example.
5. Define a linear continuous.
6. Let $X = \{a, b, c\}$ and $J = \{\phi, x, \{a\}\}$. Is x connected with respect to J? Justify your answer.
7. Is every closed interval in $\mathbb{R}$ compact? Justify.
8. Define locally compact. Give an example.
9. Whether the space $\mathbb{R}_k$ is Hausdorff or not? Justify your answer.
10. State the Urysohn lemma.

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) If $\mathcal{B}$ is a basis for the topology of x and $\mathcal{C}$ is a basis for the topology of y, then prove that the collection $\mathcal{D} = \{B \times C / B \in \mathcal{B}$ and $C \in \mathcal{C}\}$ is a basis for the topology of $x \times y$.

Or

(b) If A is a subspace of X and B is subspace of y, then prove that the product topology on $A \times B$ is the same as the topology $A \times B$ inherits as a subspace of $x \times y$.

12. (a) State the prove the pasting lemma.

Or

(b) State and prove uniform limit theorem.

13. (a) Prove that the image of a connected space under a continuous map is connected.

Or

(b) Show that a space x is locally connected if an only if for every open set U of x, each component of U is open in x.

14. (a) State and prove uniform continuity theorem.

Or

(b) Let x be locally compact Haudorff and let A be a subspace of x. If A is closed in x or open in x then prove that A is locally compact.

15. (a) Prove that every compact Hausdorff space is normal.

Or

(b) When will you say that a space is said to be completely regular? Also prove that a subspace of a completely regular space is completely regular.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. (a) Let A be a subset of the topological space x and let A' be the set of all limit points of A. Prove that $\overline{A} = A \cup A'$.

(b) Show that every finite point set in a Hausdorff space x is closed.

17. Let $\overline{d}\,(a,b) = \min\{|a-b|, 1\}$ be the standard bounded metric on $\mathbb{R}$. If x and y are two points of $\mathbb{R}^{w}$, define $D(x,y) = \sup\left\{\dfrac{\overline{d}(x_i, y_i)}{i}\right\}$. Prove that D is a metric that induces the product topology on $\mathbb{R}^{w}$.

18. Let L is a linear continuous in the order topology, then L is connected and so are intervals and rays in L.

19. Show that the product of finitely many compact spaces is compact.

20. State and prove the Tietze extension theorem.

S–6312

Sub. Code
23MMA3C4

M.Sc. DEGREE EXAMINATION, APRIL 2025

Third Semester

Mathematics

INDUSTRIAL STATISTICS

(CBCS – 2023 onwards)

Time : 3 Hours   Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define biased and unbiased estimator. Give an example.

2. What is meant by a composite statistical hypothesis?

3. State the properties of sufficient statistic.

4. Define complete family of probability density function.

5. Write short notes on Bayes confidence interval.

6. Define asymptotically efficient.

7. What do you mean by a uniformly most powerful test?

8. Define likelihood ratio test.

9. Define a noncentral chi-square distribution.

10. State the assumptions of analysis of variance.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Let $\overline{X}$ be the mean of a random sample of size n from a distribution that is $N(\mu, 9)$. Find n such that $P_r(\overline{X} - 1 < \mu < \overline{X} + 1) = 0.90$, approximately.

Or

(b) Let two independent random samples, each of size 10, from two normal distributions $N(\mu, \sigma^2)$ and $N(\mu_2, \sigma^2)$ yield $\overline{x} = 4.8$, $s_1^2 = 8.64$, $\overline{y} = 5.6$, $s_2^2 = 7.88$. Find a 95% confidence interval for $\mu_1 - \mu_2$.

12. (a) Let $X_1, X_2, \ldots, X_n$ denote a random sample from a normal distribution with mean zero and variance θ, $0 < \theta < \infty$. Show that $\sum_1^n x_i^2 / n$ is an unbiased estimator of θ and variance $2\theta^2/n$.

Or

(b) Let $X_1, X_2, \ldots, X_n$ denote a random sample from a distribution that is $N(\theta, 1)$, $-\infty < \theta < \infty$. Find the unbiased minimum variance estimator of θ^2.

13. (a) With the usual notations, show that the fisher information $I(\theta) = \int_{-\infty}^{\infty} \left[\frac{\partial \ln f(x\ ;\ \theta)}{\partial \theta} \right]^2 f(x\ ;\ \theta)\, dx$.

Or

(b) Let $X_1, X_2, \ldots, X_n$ denote a random sample from the distribution $N(\theta, 1)$, $-\infty < \theta < \infty$. Find the maximum likelihood estimate $\hat{\theta}$ of θ.

14. (a) Show that the likelihood ratio test leads to Neyman Pearson theorem for testing $H_0 : \theta = \theta_0$ against $\theta_1 : \theta = \theta_1$ where θ_0 and θ_1 are fixed numbers.

Or

(b) Explain the concept of sequential probability ratio test.

15. (a) State and prove the Boole's inequality.

Or

(b) Enumerate the following terms :

(i) Least squares method;

(ii) Correlation coefficient.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Let x have a p.d.f of the form $f(x\,;\theta)=\theta x^{\theta-1}, 0<x<1$, zero, elsewhere, where $\theta\in\{\theta:\theta=1,2\}$. To test the simple hypothesis $H_0:\theta=1$ against the alternative simple hypothesis $H_1:\theta=2$, use a random sample x_1, x_2 of size $n=2$ and define the critical region to be $C=\{(x_1,x_2):3/4\le x_1x_2\}$. Find the power function of the test.

17. (a) State and prove the Rao-Blackwell theorem.

    (b) Let $\overline{X}$ denote the mean of the random sample $X_1, X_2,\ldots,X_n$ from a gamma-type distribution with parameters $\alpha>0$ and $\beta=\theta>0$. Compute $E[x_1/\overline{x}]$.

18. Let $X_1, X_2,\ldots,X_n$ denote a random sample from a distribution that is $b(1,\theta)$, $0<\theta<1$. Find the Baye's solution δ.

19. State and prove the Neyman-Pearson theorem.

20. Let $X_{1j}, X_{2j},\ldots,X_{ajj}$ represent independent random samples of size a_j from normal distributions with means μ_j and variance σ^2, $j=1,2,\ldots,b$. Show that

$$\sum_{j=1}^{b}\sum_{i=1}^{a_j}\left(X_{ij}-\overline{X}..\right)^2=\sum_{j=1}^{b}\sum_{i=1}^{a_j}\left(X_{ij}-\overline{X}._j\right)^2+\sum_{j=1}^{b}a_j\left(\overline{X}._j-\overline{X}..\right)^2,$$

Here $\overline{x}..=\sum_{j=1}^{b}\sum_{i=1}^{a_j}X_{ij}/\sum_{j=1}^{b}a_j$ and $\overline{X}._j=\sum_{i=1}^{a_i}X_{ij}/a_j$.

S–6313

Sub. Code
23MMA3E1

M.Sc. DEGREE EXAMINATION, APRIL 2025

Third Semester

Mathematics

Elective – ALGEBRAIC NUMBER THEORY

(CBCS – 2023 onwards)

Time : 3 Hours　　　　　　　　　　　　　　　　　　　Maximum : 75 Marks

Part A　　　　　　　　　　(10 × 2 = 20)

Answer **all** questions.

1. Define principal ideal.

2. Find the degrees of the field extensions $Q\left(\sqrt{7}\right) : Q$.

3. Express $Q\left(\sqrt{2}, \sqrt[3]{5}\right)$ in the form $Q(\theta)$.

4. Find the discriminant of $Q\left(\sqrt{3}, \sqrt{5}\right)$.

5. Define norm.

6. Compute integral bases $Q\left(\sqrt{2}, \sqrt{3}\right)$.

7. Define a cyclotomic field.

8. Define a quadratic field.

9. What is trivial factorizations?

10. State Fermat theorem.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) It $p \in z[t]$ and its image $\overline{p} \in z_n[t]$ is irreducible, with $r\,\overline{p} = rp$, then prove that p is irreducible as an element of $z[t]$.

Or

(b) Let R be a ring. Then prove that every symmetric polynomial in $R[t_1, t_2, \dots t_n]$ is expressible as a polynomial with coefficients in R in the elementary symmetric polynomials $\delta_1, \delta_2, \dots \delta_n$.

12. (a) Show that the set A of algebraic number is a subfield of the complex field C.

Or

(b) Show that an algebraic number α is an algebraic integer iff its minimum polynomial ever Q has coefficients in Z.

13. (a) Suppose $\alpha_1, \alpha_1, \dots \alpha_n \in D$ form a Q-basis for K. If $\Delta[\alpha_1, \alpha_2, \dots \alpha_n]$ is square free then prove that $\{\alpha_1, \alpha_2, \dots \alpha_n\}$ is an integral basis.

Or

(b) Find the ring of integers of $Q\left(\sqrt[3]{5}\right)$.

14. (a) Show that the quadratic fields are precisely those of the form $Q\left(\sqrt{d}\right)$ for d a square free rational integer.

Or

(b) Prove that the discriminant of $Q(\zeta)$ where $\zeta = e^{2\pi i/p}$ and p is an odd prime is $(-1)^{(p-1)/2}.p^{p-2}$.

15. (a) If D is a domain and x, y are non-zero elements of D then prove that

(i) x is a unit iff $\langle x \rangle = D$

(ii) x is irreducible iff $\langle x \rangle$ is maximal among the proper principal ideals of D.

Or

(b) If a domain D is noetherian, then prove that factorization into irreducibles is possible in D.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. Let G be a free abelian group of rank r and H a subgroup of G. The prove that G/H is finite iff the ranks of G and H are equal.

17. Show that a complex number θ is an algebraic integer iff the additive group generated by all powers $1, \theta, \theta^2, \ldots$ is finitely generated.

18. Show that every number filed K possess an integral basis and the additive group of D is free abelian of rank n equal to the degree of K.

19. Prove that the ring D of integers of $Q(\zeta)$ is $Z[\zeta]$.

20. Let D be the ring of integers in a number field K and let $x, y \in D$. Then prove that

(a) x is a unit iff $N(x) = \pm 1$

(b) If x and y are associates, then $N(x) = \pm N(y)$

(c) If $N(x)$ is a rational prime, then x is irreducible in D.

S–6315

Sub. Code
23MMA3E3

M.Sc. DEGREE EXAMINATION, APRIL 2025

Third Semester

Mathematics

Elective — STOCHASTIC PROCESSES

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Write down the properties of auto covariance function.

2. Define a Markov chain.

3. Define persistent.

4. State the general Erogodic theorem.

5. What is meant by pure birth process?

6. Define transition density matrix.

7. Define a renewal process.

8. State Blackwell's theorem.

9. Prove that for $r, n = 0,1,2\ldots\ E\{x_n + r | x_n\} = X_n m^r$.

10. Define a Markov branching process.

Part B $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) Explain the correlated random walk.

Or

(b) If the process $X(t) = A\cos wt + B\sin wt$, where A, B are uncorrelated random variables with mean 0, variance 1 and w is a positive constant. Then prove that $\{x(t), t \geq 0\}$ is covariance stationary.

12. (a) If state K is persistent null, then prove that for every j, $\lim_{n\to\infty} P_{jk}^{(n)} \to 0$.

Or

(b) If state j is persistent, then prove that for every state k that can be reached from state j, $F_{kj} = 1$.

13. (a) If $\{N(t)\}$ is a Poisson process and $s < t$, then prove that $P\{N(s) = k(|N(t) = n\} = \binom{n}{k}(S/t)^k [1-(s/t)]^{n-k}$.

Or

(b) Prove that the p.g.f of a non-homogeneous process $\{N(t), t \geq 0\}$ is given by $Q(S,t) = \exp\{m(t)\,(s-1)\}$ where $m(t) = \int_0^t \lambda(x)\,dx$ is the expectation of $N(t)$.

14. (a) Show that the renewal function M satisfies the equation $M(t)=F(t)+\int_0^t M(t-x)\, d\, F(x)$.

Or

(b) State and prove Wald's equation.

15. (a) Prove that for a Galton-Watson process with $m=1$ and $\sigma^2<\infty$, $\lim_{n\to\infty}\frac{1}{n}\left\{\frac{1}{1-P_n(s)}-\frac{1}{1-s}\to\frac{\sigma^2}{2}\right\}$ uniformly in $0\le\delta<1$.

Or

(b) If $m=1, \sigma^2<\infty,$ then Prove that

(i) $\lim_{n\to\infty} n\, p\{x_n>0\}=\frac{2}{\sigma^2}$

(ii) $\lim_{n\to\infty} E\left\{\frac{x_n}{n}\,\middle|X_n>0\right\}=\frac{\sigma^2}{1}$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Derive polya's urn model.

17. State and prove ergodic theorem.

18. Prove that the postulates for Poisson process $N(t)$ follows Poisson distribution with mean λt that is $P_n(t)$ is given by the Poisson law $P_n(t)=\frac{e^{-\lambda t}(\lambda t)^n}{n!}$, $n=0,1,2,...$

19. State and prove Elementary renewal theorem.

20. State and prove Yaglom's theorem.

S–6316

Sub. Code
23MMA4C1

M.Sc. DEGREE EXAMINATION, APRIL 2025

Fourth Semester

Mathematics

FUNCTIONAL ANALYSIS

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A ($10 \times 2 = 20$)

Answer **all** questions.

1. Define a normed linear space. Give an example.

2. What is meant by continuous linear trnsformation?

3. Define the conjugate space of hilbert space.

4. Define the following terms:

   (a) Normal operator;

   (b) unitary operator

5. Define total matrix algebra of degree n.

6. What is meant by the spectrum of T?

7. Define a breach algebra. Give an example.

8. If r is an element of R, then prove that $1-xr$ is regular for every x.

9. What is meant by the gelfand mapping?

10. When will you stay that f is said to be a * – isomorphism?

Part B $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) Let p be a real number such that $1 \leq p < \infty$ and l_p^n denote the linear space of all n–tuples of scalars with the norm of a vector space $x = (x_1, x_2 ...... x_n)$ defined by $\|x\|_p = \left(\sum_{i=1}^{n} |x_i|^p \right)^{\frac{1}{p}}$. Show that l_p^n is a banach space.

Or

(b) State and prove the Hahn–Banach theorem.

12. (a) Let H be the Hilbert space. Prove that the adjoint operation $T \to T^*$ on $B(H)$ has the following properties.

(i) $(T_1 + T_2)^* = T_1^* + T_2^*$;

(ii) $(T_1 T_2)^* = T_2^* T_1^*$;

(iii) $T^{**} = T$

(iv) $\|T^*\| = \|T\|$

Or

(b) If T is an operator on H, then prove that T is normal if and only if its real and imaginary parts commute.

13. (a) Let B be a basis for H and T an operator whose matrix relative to B is $[\alpha_{ij}]$. Prove that T is non–singular if and only if $[\alpha_{ij}]$ is non–singular and in this case $[\alpha_{ij}]^{-1} = [T^{-1}]$.

Or

(b) Let T be on operator on H. Prove that following:

(i) T is singular if and only if $o \in \sigma(T)$;

(ii) If T is non–singular, then $\lambda \in \sigma(T)$ if and only if $\lambda^{-1} \in \sigma(T^{-1})$.

14. (a) Prove that every element x for which $\|x - 1\| < 1$ is regular, and the inverse of such an element is given by the formula $x^{-1} = 1 + \sum_{n=1}^{\infty} (1 - x)^n$.

Or

(b) If I is a proper closed two–sided ideal in A, then prove that the quotient algebra A/I is a banach algebra.

15. (a) If f_1 and f_2 are multiplicative functionals on A with the same null space M, then prove that $f_1 = f_2$.

Or

(b) If x is a normal element in a B^*–algebra, then prove that $\|x^2\| = \|x\|^2$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. State and prove that open mapping theorem.

17. (a) State and prove the schwarz inequality.

    (b) State and prove pythagorean theorem.

18. State and prove the spectral theorem.

19. With the usual notations, prove that

    (a) $\sigma(x)$ is non–empty

    (b) $\sigma(x^n) = \sigma(x)^n$

20. If A is a commutative B^*– algebra, then prove that the gelfand mapping $x \to \hat{x}$ is an isometric * – isomorphism of A onto the commutative B^*– algebra $C(\mathcal{M})$.

S–6317

Sub. Code
23MMA4C2

M.Sc. DEGREE EXAMINATION, APRIL 2025

Fourth Semester

Mathematics

DIFFERENTIAL GEOMETRY

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define the following terms :

   (a) Osculating plane

   (b) Radius of curvature.

2. Define an involute and an evolute.

3. Define the anchor ring.

4. Define isometric surfaces.

5. Write down a characteristics property of a geodesic.

6. What is the difference between the circumstances of a geodesic circle of small radius r and the Euclidean circumstances approximately? Justify.

7. State the Meusnier's theorem.

8. Define the following terms :

(a) Characteristics line

(b) Polar developable.

9. State the Hilbert's theorem.

10. Define a metric $\rho(A,B)$ on a connected surface and explain why $\rho(A,B)=0 \Leftrightarrow A=B$?

Part B $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) Find the length of the curve given as the intersection of the surfaces $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, $x=a\cosh\left(\frac{z}{a}\right)$, from the point $(a,0,0)$ the point (x,y,z).

Or

(b) Define Bertran's curves. Also prove that the tangents to γ and γ_1 are inclined at a constant angle.

12. (a) Find the coefficients of the direction which makes an angle $\frac{\pi}{2}$ with the direction whose coefficients are (l,m).

Or

(b) Show that the metric is invariant under a parameter transformation.

13. (a) Prove that on the general surface, a necessary and sufficient condition that the curve $v = c$ be a geodesic is $EE_2 + FE_1 - 2EF_1 = 0$ when $v = c$, for all values of u.

Or

(b) State and prove the Gauss-Bonnet theorem.

14. (a) Derive the second fundamental form of a surface.

Or

(b) Show that a necessary and sufficient condition that a curve on a surface be a line of curvature is that the surface normals along the curve form a developable.

15. (a) Prove that the only compact surface with constant Gaussian curvature and spheres.

Or

(b) State and prove the Jacobi theorem.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Establish the Serret-Frenet formulae.

17. (a) On the paraboloid $x^2 - y^2 = z$, find the orthogonal trajectories of the sections by the planes z=constant.

(b) If θ is the angle at the point (u,v) between the two directions given by the differential equation $Pdu^2 + 2Q\, du\, dv + Rdv^2 = 0$, then prove that

$$\tan\theta = \frac{2H\left(Q^2 - PR\right)^{1/2}}{ER - 2FQ + GP}.$$

18. Derive the Liouville's formula for the geodesic curvature of a curve.

19. State and prove the Rodrigue's formula.

20. State and prove the Hilbert's lemma.

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S–6318

Sub. Code
23MMA4C3

M.Sc. DEGREE EXAMINATION, APRIL 2025

Fourth Semester

Mathematics

MECHANICS

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Write down the holonomic and non-holonomic constraints.

2. State the principle of virtual work.

3. Write down the Lagrangian for a particle moving under gravity.

4. When will you say that a conservative system is a natural system?

5. State the multiplier rule for the analysis of the stationary values of $\int_{t_0}^{t_1} L dt$.

6. Define the Legendre transformation.

7. State the Hamilton's principle function.

8. State the Stackel's theorem.

9. Show that the identity transformation is a canonical transformation.

10. Define "Paint transformation" and "Momentum transformation".

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) State and prove the D' Alembert's principle.

Or

(b) Prove with usual notations that the rotational kinetic energy can be written in the form $T_{rot} = \frac{1}{2} w^T I w$.

12. (a) Find the differential equations for a spherical pendulum of length l.

Or

(b) Discuss the Kepler problem using an ignorable coordinates.

13. (a) Find the solution of the Kepler problem using Hamiltonian H.

Or

(b) State the principle of least action. Obtain the Jacobi's form of the principle of least action.

14. (a) What do you mean by Pfaffian differential forms? Also derive the Hamilton's canonical equations.

Or

(b) Define Liouville's system and establish that the Liouville's conditions are sufficient for the separability of an orthogonal system.

15. (a) Prove that the transformation $Q = \sqrt{2q}\, e^{t} \cos p$, $P = \sqrt{2q}\, e^{-t} \sin p$ is canonical and obtain its generating function.

Or

(b) Obtain the bilinear covariant associated with the Pfaffian differential form.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) Explain the following terms :

(i) Degrees of freedom

(ii) Generalized coordinates

(iii) Configuration space

(iv) Virtual displacement

(b) Two frictionless blocks of equal mass m are connected by a massless rigid rod. Using x_1 and x_2 as coordinates, solve for the force F_2 if the system is in static equilibrium.

17. Obtain the Lagrange's equations for a holonomic system in the form $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial \dot{q}_i} = 0,\ i = 1, 2, \ldots, n$.

18. Derive the Euler-Lagrange equation in the form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$.

19. Derive the modified Hamilton - Jacobi equation in the form $H\left(q, \frac{\partial w}{\partial q}\right) = \alpha_n$. Discuss the case of conservative system with ignorable coordinates, using the modified Hamilton-Jacobi equation.

20. Define a homogeneous canonical transformation and analyse the generating functions associated with such a transformation.

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S–6319

Sub. Code
23MMA4E1

M.Sc. DEGREE EXAMINATION, APRIL 2025

Fourth Semester

Mathematics

Elective – ADVANCED NUMERICAL ANALYSIS

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. What is meant by iteration function?

2. Define asymptotic error constant.

3. When will iteration method succeed?

4. Define pivot elements.

5. What is meant by finite elements and knots?

6. What is a natural spline?

7. Define error of approximation.

8. What is meant by abscissas?

9. Give the multistep methods available for solving ordinary differential equation.

10. What is the truncation error of Taylor's method?

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Perform three iterations of the multipoint iteration method (2.33), to find the root of the equation $f(x) = \cos x - xe^x = 0$.

Or

(b) Find the number of real and complex roots of the polynomial equation $P_3(x) = x^3 - 5x + 1 = 0$ using Sturm sequences.

12. (a) For the matrix $A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 5 & 2 \\ 2 & 2 & 3 \end{bmatrix}$

(i) Find all the eigen values and the corresponding eigenvectors.

(ii) Verify S^{-1} AS is a diagonal matrix, where S is the matrix of eigenvectors.

Or

(b) Determine the condition number of the matrix $A = \begin{bmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{bmatrix}$ using the maximum absolute row sum norm.

13. (a) Obtain the piecewise linear interpolating polynomials for the function $f(x)$ defined by the data

x	1	2	4	8
$f(x)$	3	7	21	73

Hence, estimate the value of $f(3)$ and $f(7)$.

Or

(b) Given the data

x	0	1	2	3
$f(x)$	1	2	33	244

fit quadratic splines with $M(0) = f''(0) = 0$. Hence, find an estimate of f (2.5).

14. (a) Find the approximate value of $\int_0^1 \frac{\sin x}{x} dx$ using

(i) mid-point rule and

(ii) two-point open type rule.

Or

(b) Find the approximate value of

$I = \int_0^1 \frac{dx}{1+x}$, using Simpson's rule.

15. (a) Find singlestep methods for the differential equation $y' = f(t, y)$, which produce exact results for $y(t) = a + b\cos t + c\sin t$.

Or

(b) Solve the initial value problem $u' = -2tu^2, u(0) = 1$ using the mid-point method, with h = 0.2, over the interval [0, 1]. Use the Taylor series method of second order to compute $u(0.2)$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Obtain the complex roots of the equation $f(z) = z^3 + 1 = 0$ correct to eight decimal places. Use the initial approximation to a root as $(x_0, y_0) = (0.25, 0.25)$. Compare with the exact values of the roots $(1 + i\sqrt{3})/2$.

17. Find all the eigenvalue of the matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 2 \\ -1 & 2 & 1 \end{bmatrix}$ using the Jacobi method. Iterate till the off-diagonal elements, in magnitude, are less than 0.0005.

18. Obtain the cubic spline approximation for the function defined by the data

x	0	1	2	3
$f(x)$	1	2	33	244

with $M(0) = 0 M(3) = 0$. Hence, find an estimate of $f(2.5)$.

19. Find the quadrature formula $\int_0^1 f(x) \frac{dx}{\sqrt{x(1-x)}} = \alpha_1 f(0) + \alpha_2 f\left(\frac{1}{2}\right) + \alpha_3 f(1)$, which is exact for polynomials of highest possible degree. Then use the formula on $\int_0^1 \frac{dx}{\sqrt{x - x^3}}$ and compare with the exact.

20. Given the initial value problem $u' = 2tu^2, u(0) = 1$ estimate $u(0.4)$ using

(a) modified Euler-Cauchy method and

(b) Heun method, with h = 0.2. Compare the results with the exact solution $u(t) = \frac{1}{(1+t^2)}$.

S–6320

Sub. Code
23MMA4E2

M.Sc. DEGREE EXAMINATION, APRIL 2025.

Fourth Semester

Mathematics

Elective : ALGEBRAIC TOPOLOGY

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define path homotopy. Give an example.

2. Define a covering map.

3. When will you say that A is said to be deformation retract of X?

4. Define projective n-space.

5. If $G = G_1 \oplus G_2$, then prove that G/G_2 is isomorphic to G_1.

6. What is meant by the external free product of the groups G_x?

7. Define the homology groups of x.

8. Define triangulation of x.

9. What is meant by a universal covering space?

10. Define a regular covering map. Give an example.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) With the usual notations, prove that the relation $\simeq$ and $\simeq_p$ are equivalence relations.

Or

(b) State and prove the Brouwer fixed-point theorem for the disc.

12. (a) Let $f : x \to y$ be continuous and let $f(x_0) = y_0$. If f is a homotopy equivalence, then prove that $f_* : \pi_1(x, x_0) \to \pi_1(y, y_0)$ is an isomorphism.

Or

(b) Show that the fundamental group of the double torus is not abelian.

13. (a) Let $G = G_1 * G_2$. Let N_1 be a normal subgroup of G_i, for $i = 1, 2$. If N is the least normal subgroup of G that contains N_1 and N_2 then prove that $G/N \cong (G_1 / N_1) * (G_2 / N_2)$.

Or

(b) Let x be the wedge of the circles S_α , for $\alpha \in J$. Prove that x is normal. Furthermore, any compact subspace of x is contained in the union of finitely many circles S_α.

14. (a) Let x be the space obtained from a finite collection of polygonal regions by pasting edges together according to some labelling scheme. Prove that x is a compact Hausdorff space.

Or

(b) If x is a compact connected triangulable surface, then prove that x is homeomorphic to a space obtained from a polygonal region in the plane by pasting the edges together in pairs.

15. (a) Let $p: E \to B$ and $p': E' \to B$ be covering maps. Let $p(e_0) = p'(e_0') b_0$. Prove that there is an equivalence $h: E \to E'$ such that $h(e_0) = e_0'$ if and only if the groups $H_0 = p_*(\pi_1(E, e_0))$ and $H_0' = p_*'(\pi_1(E', e_0'))$ are equal. If h exists, it is unique.

Or

(b) Find a group G of homeomorphisms of T having order 2 that T/G is homeomorphic to the Klein bottle.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Let $p: E \to B$ be a covering map and let $p(e_0) = b_0$, Prove that any path $f:[0,1] \to B$ beginning at b_0 has a unique lifting to a path $\tilde{f}$ and E beginning at e_0.

17. State and prove the fundamental theorem of algebra.

18. Given G, the subgroup $[G,G]$ is a normal subgroup of G and the quotient group $G/[G,G]$ is abelian. If $h: G \to H$ is any homomorphism from G to an abelian group H, then prove that the Kernal of h contains $[G,G]$, so h induces a homomorphism $K: G/[G,G] \to H$

19. State and prove the classification theorem.

20. Let x be path connected and locally path connected. Let G be a group of homeomorphisms of x. Prove the quotient map $\pi : X \to X / G$ is a covering map if and only if the action of G is properly discontinuous. In this case, the covering map π is regular and G is its group of covering transformations.

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